Linear System Homework 6

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Problem 1

Let A $= \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, find 3^A and A^{100000}

Solution:

$$A = E\boldsymbol{\Lambda}E^{-1} = \begin{pmatrix} -0.7071 & 0.7071 \\ 0.7071 & 0.7071 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} -0.7071 & 0.7071 \\ 0.7071 & 0.7071 \end{pmatrix}$$

$$A^{100000} = E\boldsymbol{\Lambda}^{100000}E^{-1} = \begin{pmatrix} -0.7071 & 0.7071 \\ 0.7071 & 0.7071 \end{pmatrix}\begin{pmatrix} -1^{100000} & 0 \\ 0 & 3^{100000} \end{pmatrix}\begin{pmatrix} -0.7071 & 0.7071 \\ 0.7071 & 0.7071 \end{pmatrix}$$

$$3^A = E3^{\boldsymbol{\Lambda}}E^{-1} = \begin{pmatrix} -0.7071 & 0.7071 \\ 0.7071 & 0.7071 \end{pmatrix}\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & 9 \end{pmatrix}\begin{pmatrix} -0.7071 & 0.7071 \\ 0.7071 & 0.7071 \end{pmatrix}$$

Problem 2

Which of the following functions are analytic in the open right half plane?

$\frac{s-1}{s+1}$, $\frac{s+1}{s-1}$, $\frac{1}{s^2+5s+3}$, e^{-s}

Solution:

$\frac{s-1}{s+1}$ is analytic in the open right half plan because it only has one pole at -1 , so no poles at the right plane.

To find the magnitude we use

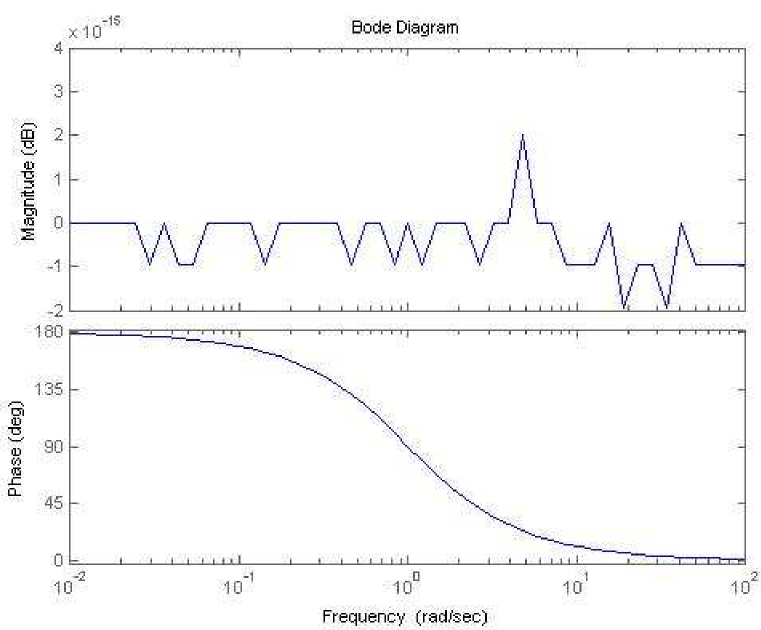
$$20\log_{10}\frac{|j\omega-1|}{|j\omega+1|} = 20\log_{10}\frac{\sqrt{\omega^2-1}}{\sqrt{\omega^2+1}} = 20\log_{10} 1 = 0$$, the graph below is multiplied by 10^{-15}

To find the phase diagram

$$\theta = \pi - \tan^{-1}\frac{w}{1} - \tan^{-1}\frac{w}{1} = \pi - 2\tan^{-1}\frac{w}{1}$$

To draw the bode plot in matlab would be

bode(tf([1,-1],[1,1]))



$\frac{s+1}{s-1}$ this is not analytic because we have a pole at 1

to find the magnitude

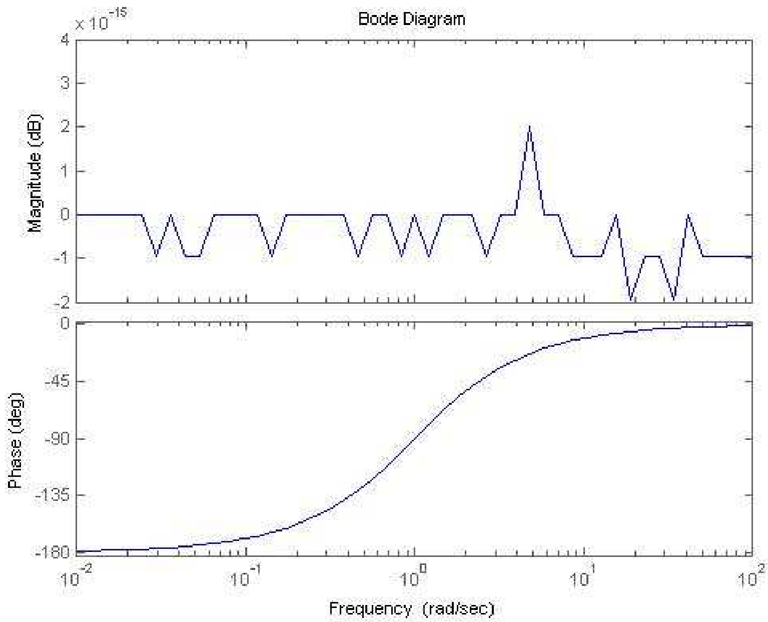
$20\ \log_{10}\frac{|j\omega+1|}{|j\omega-1|}\ = 20\ \log_{10}\frac{\sqrt{\omega^2+1}}{\sqrt{\omega^2-1}} = 20\ \log_{10} 1 = 0$, the graph below is multiplied by 10^{-15}

To find the phase diagram

$\theta = \tan^{-1}\omega - \pi + \tan^{-1}\frac{w}{1}\ \ = 2\tan^{-1}\frac{w}{1}$ - π

To draw the bode plot in matlab would be

bode(tf([1,1],[1,-1]))



$\frac{1}{s^2+5s+3}$ this is analytic because we have a poles at -0.6972 , -4.3028

to find the magnitude

$20\ \log_{10}\frac{|1|}{|5j\omega-(\omega+1)|}\ = 20\ \log_{10}\frac{1}{\sqrt{25\omega^2+(\omega+1)^2}}$

$20\ \log_{10}\frac{1}{(j\omega+0.6972)(j\omega+4.3028)}$ = 20 [-log|jω+0.6972| - log|jω+4.3028|]

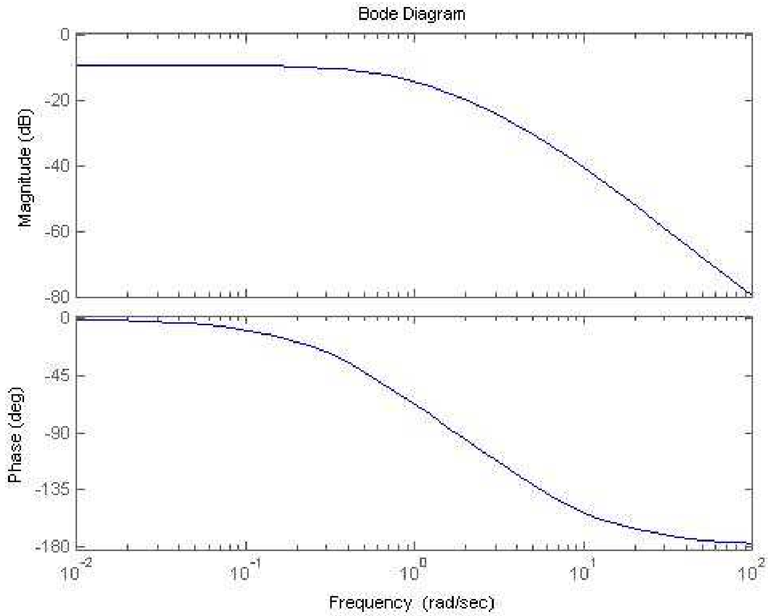
The bode plot will start at 20log $\frac{1}{(0.6972)(4.3028)}$, it will drop 20db/deca at 0.6972, and twenty more at 4.3028

To find the phase diagram

$\theta = 0$ - $\tan^{-1}\frac{w}{0.6972}$ - $\tan^{-1}\frac{w}{4.3028}$

To draw the bode plot in matlab would be

bode(tf([1],[1,5,3])



e^s this is an example of analytic everywhere, because it is differentiable at every point

the magnitude of this is $20 \log e^{-j\omega}$ or $20 \log 1 = 0$, this is just a straight line across 0, this is a rather trivial case

the phase of this is $-\omega$, this is a repeating straight line from 0 to -2π